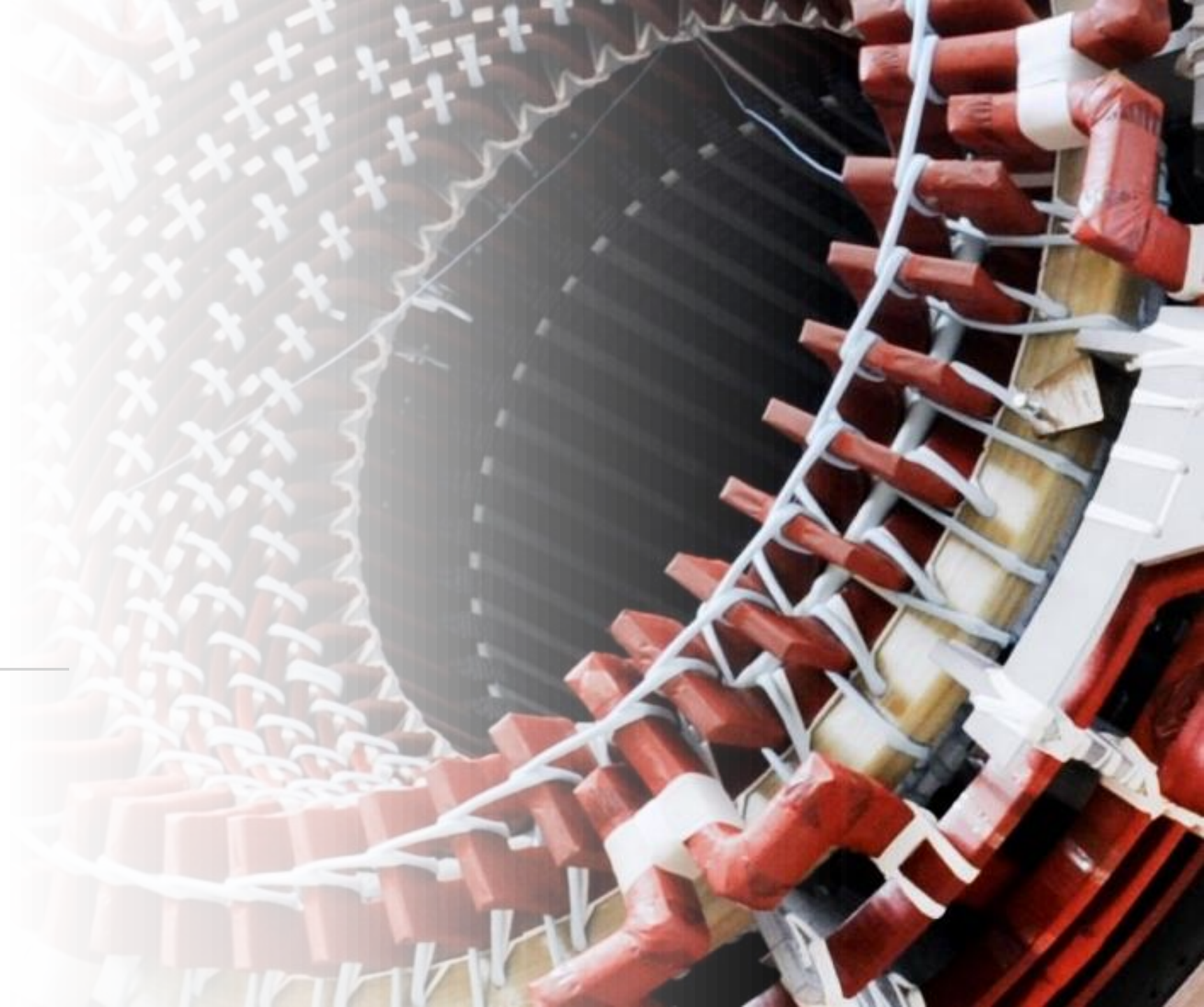




EE3124 Introduction to Electric Machines and Drives

3-DC Machines

Prof. CQ Jiang

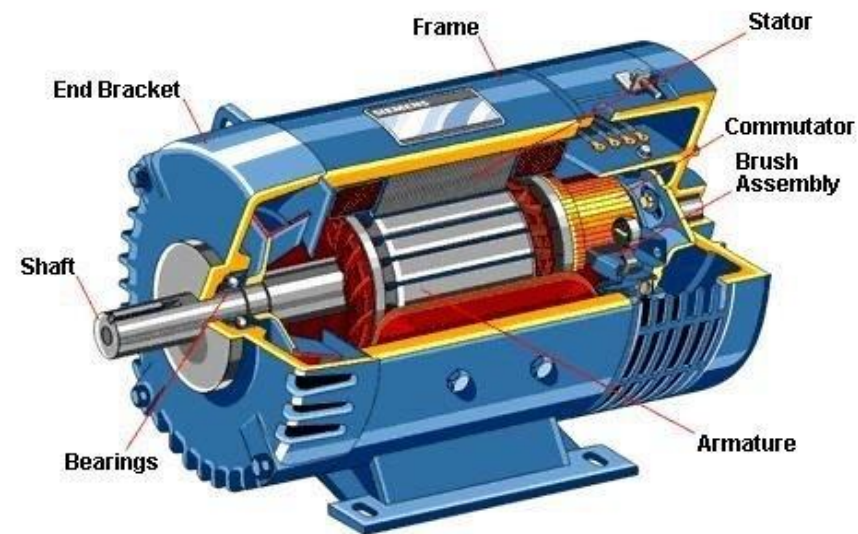
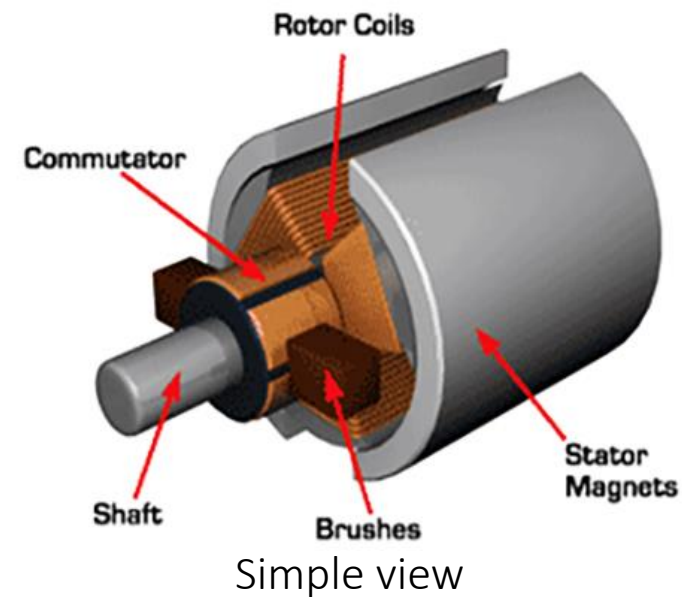


Outline

- Permanent Magnet (PM) Material
- Types of DC Machines
- Torque-speed Characteristic
- Basic Calculations of DC Motor
- Speed Control of DC Motors
- DC Motor Starter
- Losses in a DC Motor

DC Machine – Basic Construction

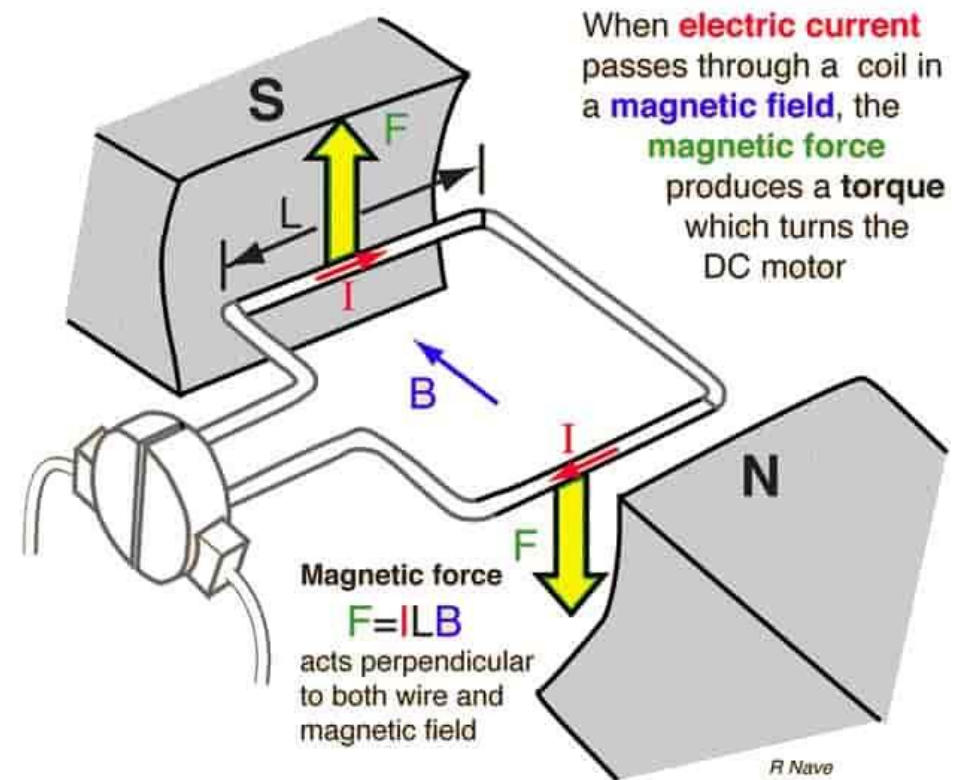
- ❑ The basic structure of a DC machine consists of the **Stator, Rotor and Commutator (Brush)**.
- ❑ The stator is the field circuit which incorporates the field winding or permanent magnets (PMs) to produce magnetic field excitation,
- ❑ whereas the rotor is the armature circuit which installs the armature winding where the armature current is bidirectional and switched by the commutator via carbon brushes.



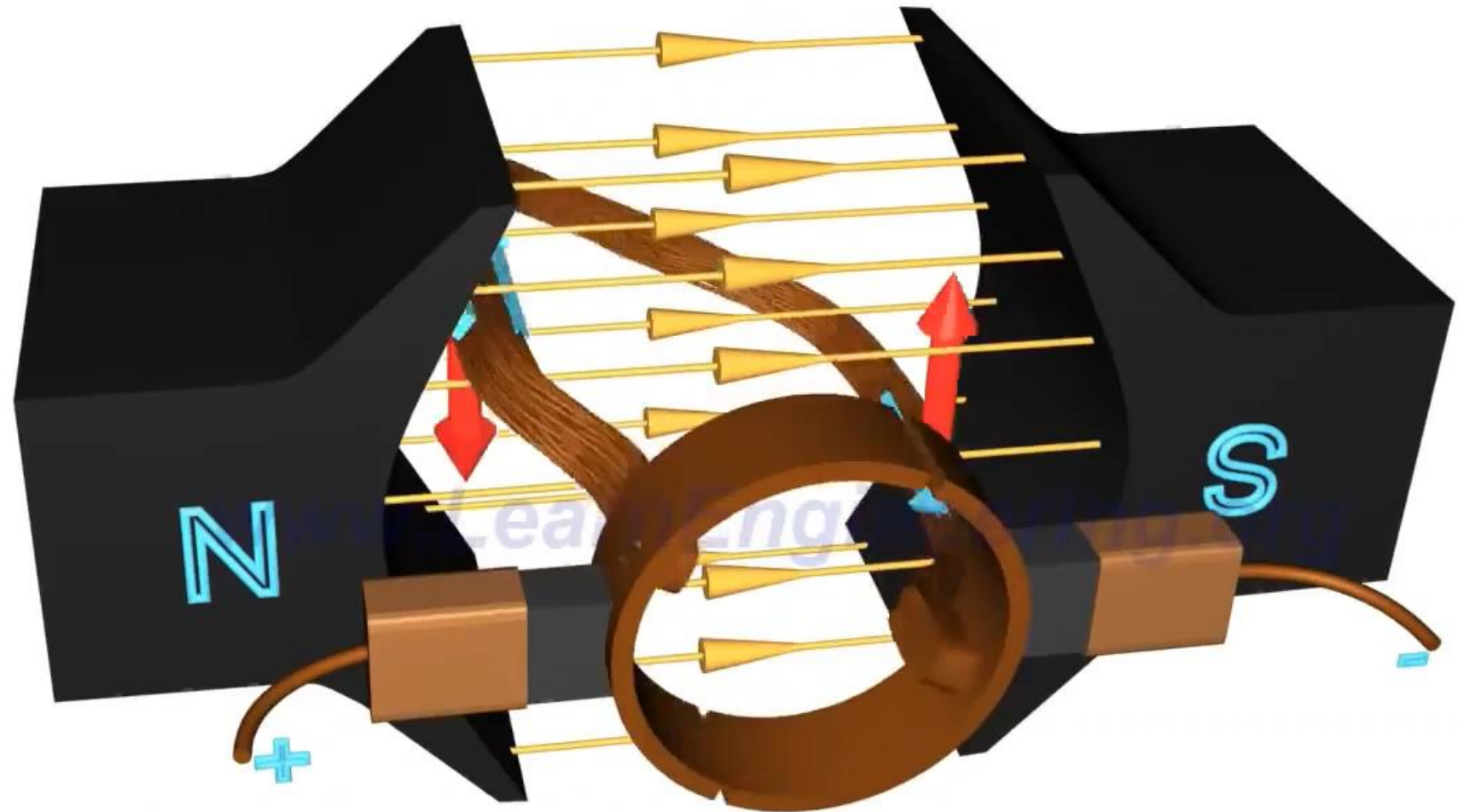
Comprehensive view

Simplified DC Machine

- It is a basic two-pole DC machine. The armature circuit consists of a simple single-turn armature coil which is connected to the DC source via a two-segment commutator and a pair of carbon brushes. The commutator serves to reverse the direction of current flow through the armature coil so that the external current always flows in only one direction; meanwhile the carbon brushes enable electrical conduction between the rotating commutator and the stationary DC source.



How does DC Motor Work?



Permanent Magnet (PM) Material

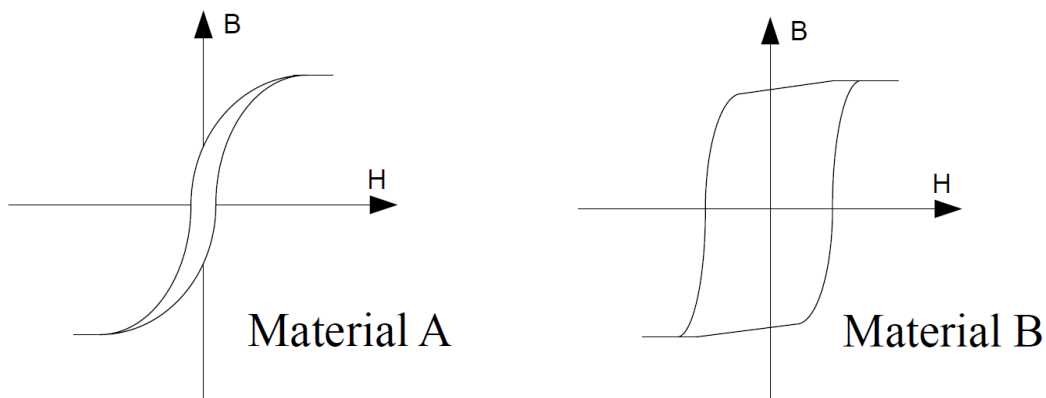
- ❑ The PMDC machine has the field being excited by permanent-magnet materials.
- ❑ Such as:
 - Neodymium-iron-boron (Nd-Fe-B)
 - Samarium-cobalt (Sm-Co)
 - Alnico (Al-Ni-Co)
 - Ferrites
- ❑ Nd-Fe-B and Sm-Co are rare-earth magnets developed after 1970s.
- ❑ Ferrites made by iron(III) oxide (Fe_2O_3) blended with barium, manganese, nickel and zinc.



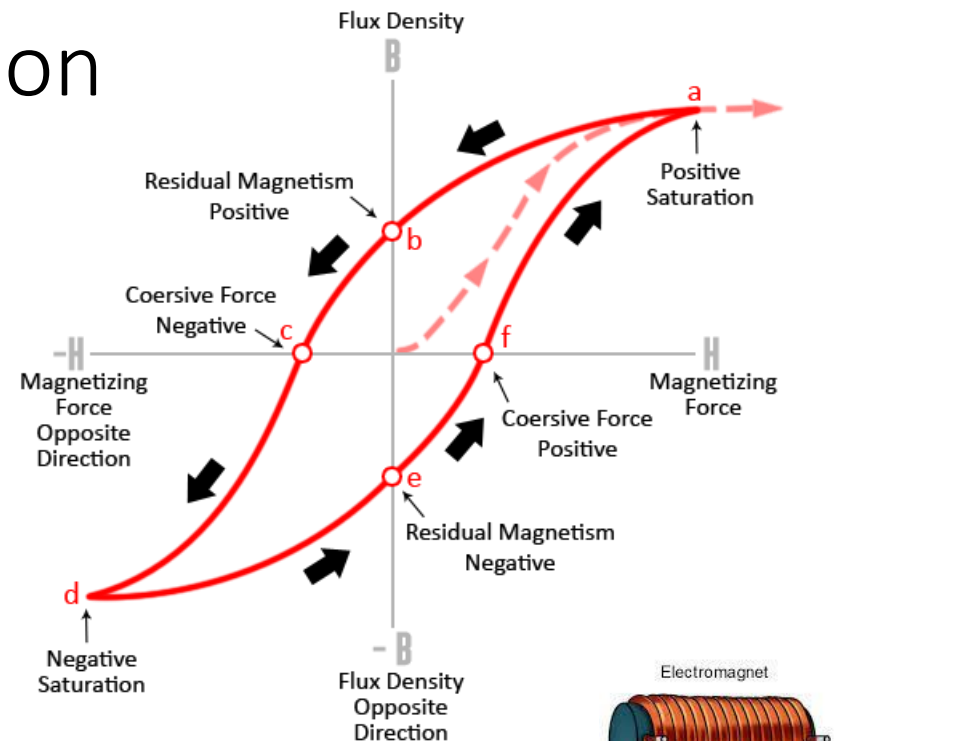
PM Magnetization and Demagnetization

A GOOD MAGNET ???

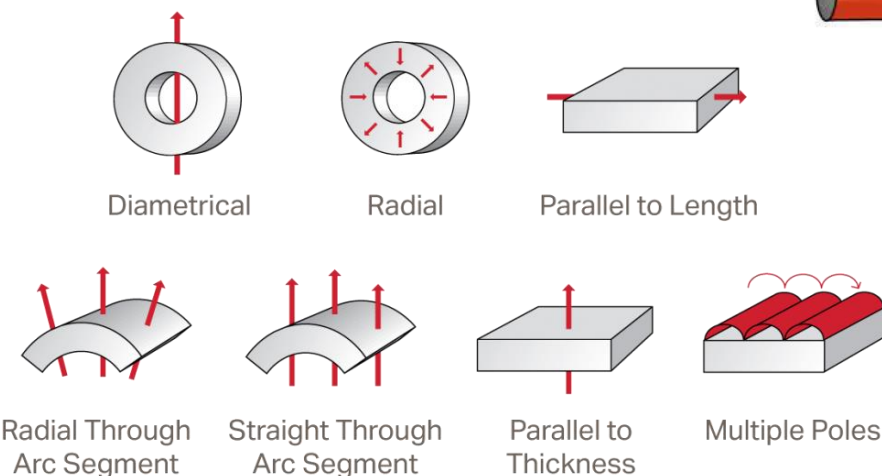
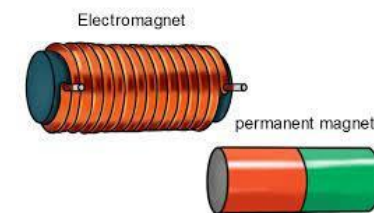
- ❑ A good material for the pole of a PMDC machine should have
 - As large a **residual flux density** B_{res} as possible, while simultaneously having as large a **coercive magnetizing intensity** H_c as possible.
 - The large B_{res} produces a large flux in the machine,
 - while the large H_c means that a very large current would be required to demagnetize the poles.



Which BH curve represents a good magnet?

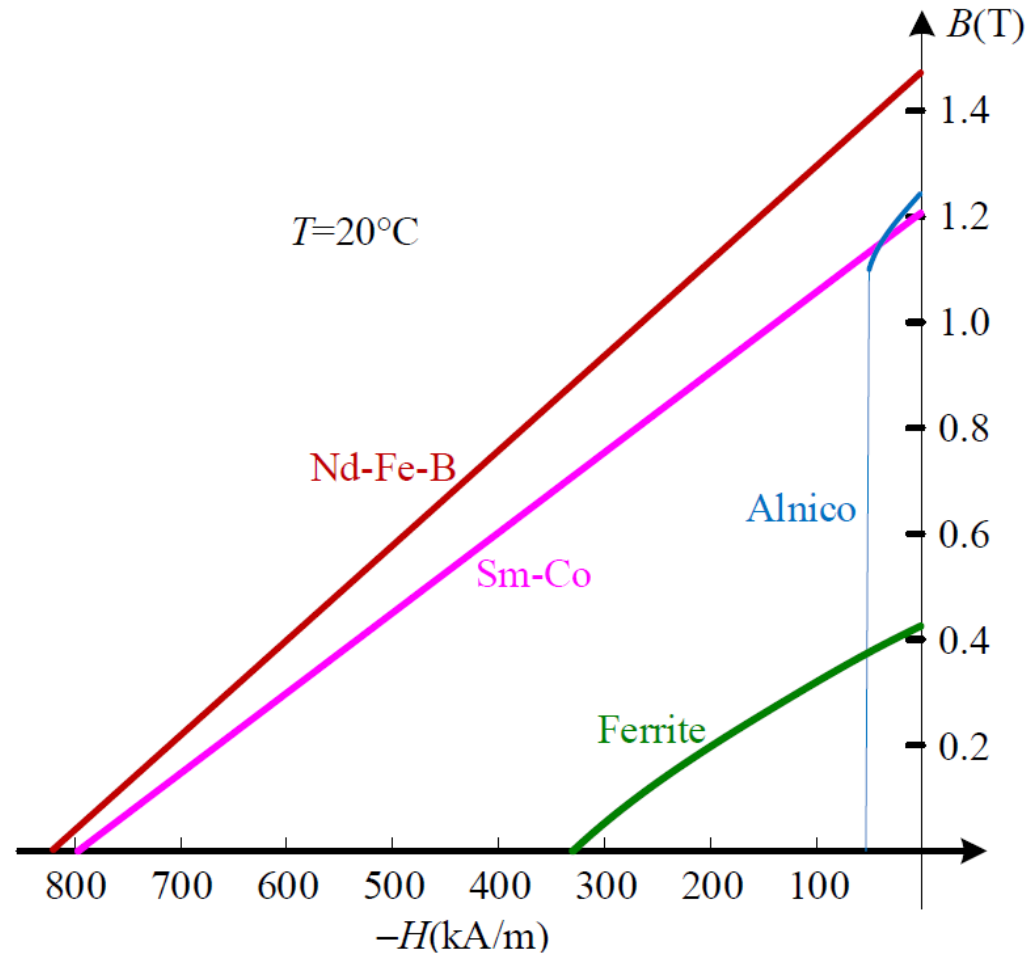


Magnetizing Patterns



PM Magnetization and Demagnetization

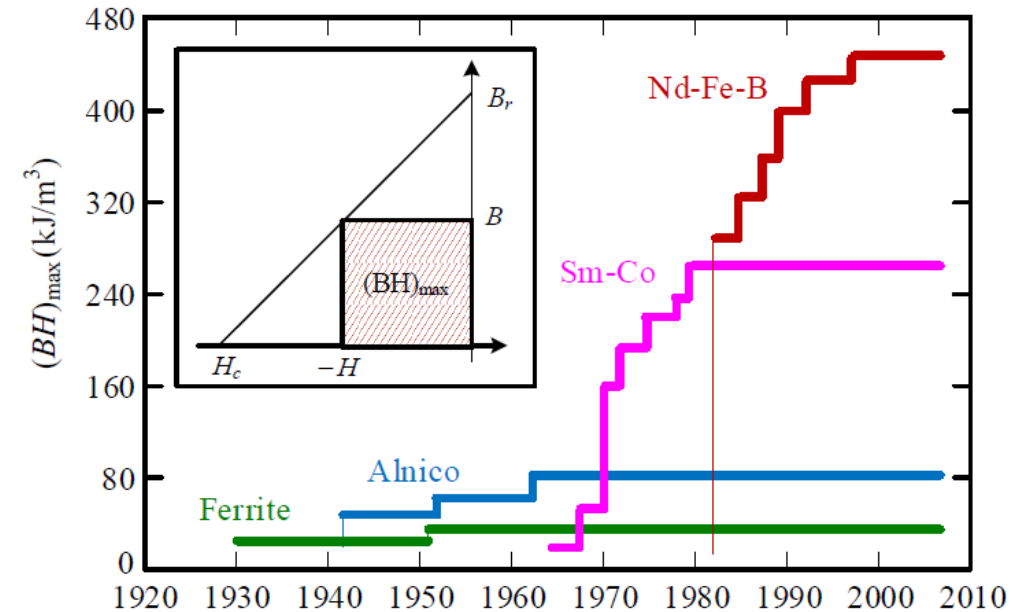
Second quadrant of the magnetization curves



PM Magnetization and Demagnetization

Maximum energy product $(BH)_{\max}$

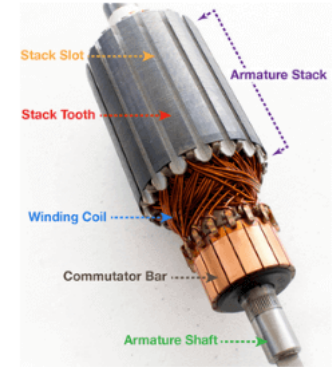
- ❑ The term **Maximum Energy Product** (BH_{\max}) is a commonly used, but often misunderstood figure of merit of magnets.
- ❑ The maximum energy product is a measurement for the maximum amount of magnetic energy stored in a magnet.
- ❑ The BH_{\max} is an Energy Density and it is oftentimes used to denote grade. The grade convention is especially used for rare earth magnets.
- ❑ The Maximum Energy Product or BH_{\max} of a magnet is an Energy Density and it is equivalent to the area of the largest rectangle that can be inscribed under the normal curve.



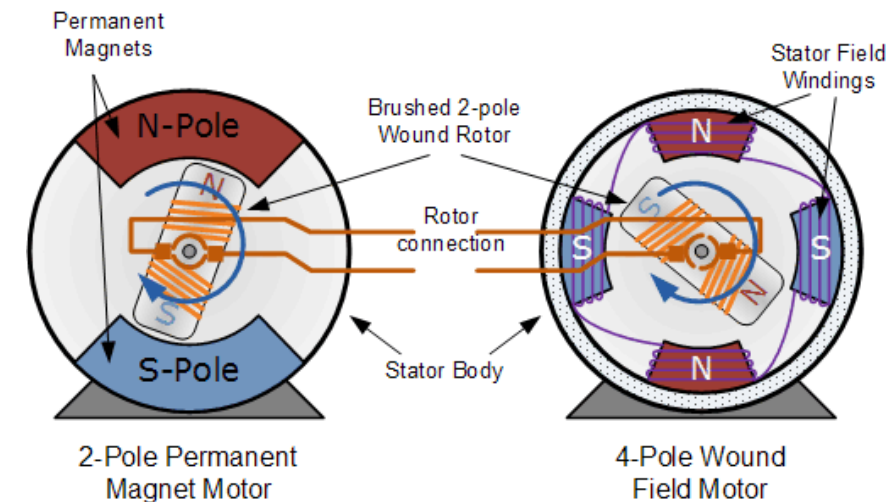
Types of DC Machines

- ❑ Different arrangements of the **field and armature** circuits create different types of DC machines, hence providing different torque-speed characteristics:
- ❑ Separately excited DC
 - series DC
 - shunt DC
 - cumulative compound DC
 - differential compound DC
- ❑ Permanent magnet DC
- ❑ The PM DC machine has relatively **higher power density and higher efficiency** than the above wound-field types because of the space-saving benefit by PMs and the absence of field losses.
- ❑ However, since the field excitation in the PM DC machine is uncontrollable, it cannot attain the operating characteristics with **flux control**.

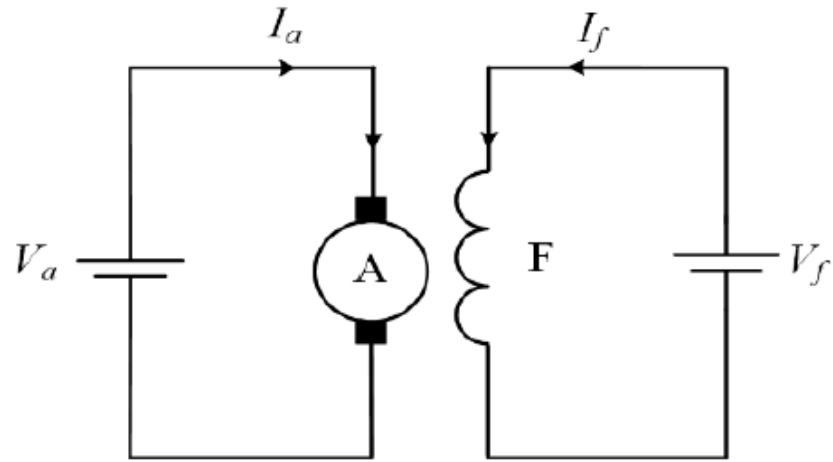
What is an Armature?



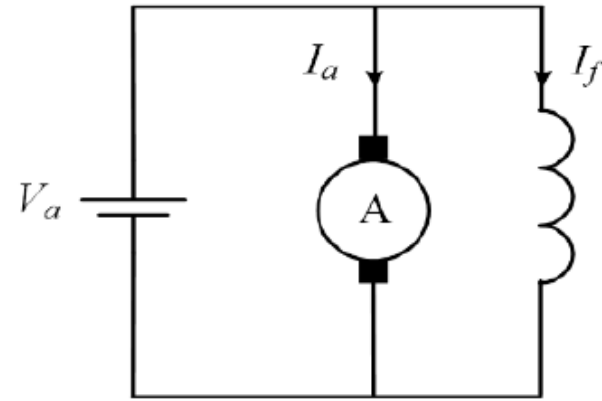
Electrical 4 U



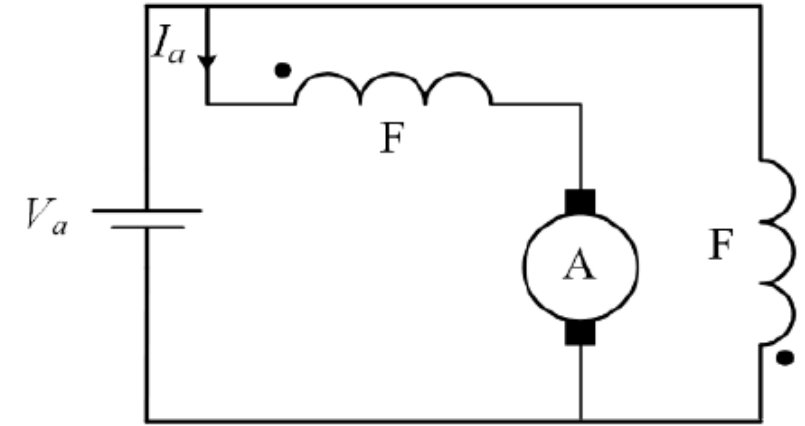
Types of DC Machines



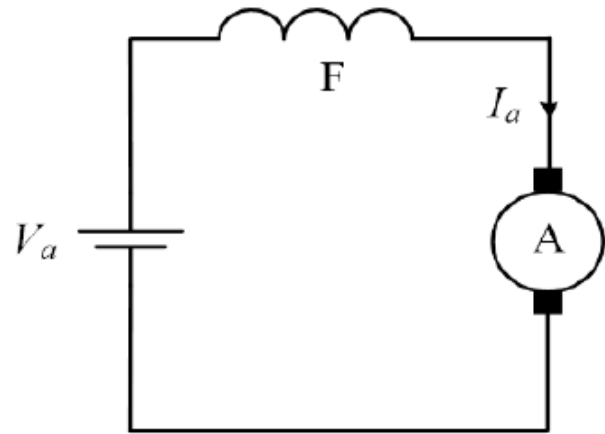
Separately excited



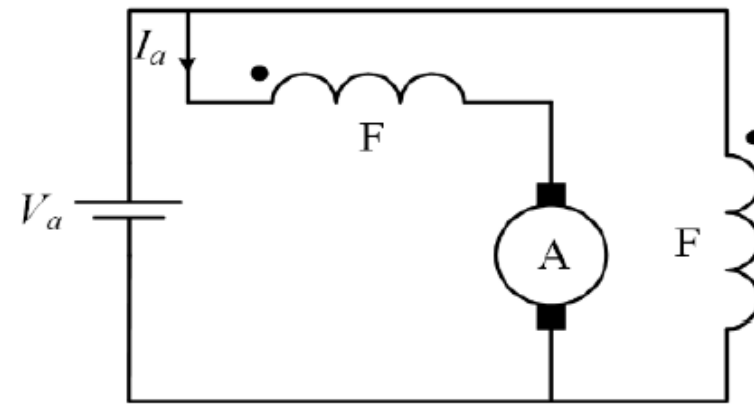
Shunt



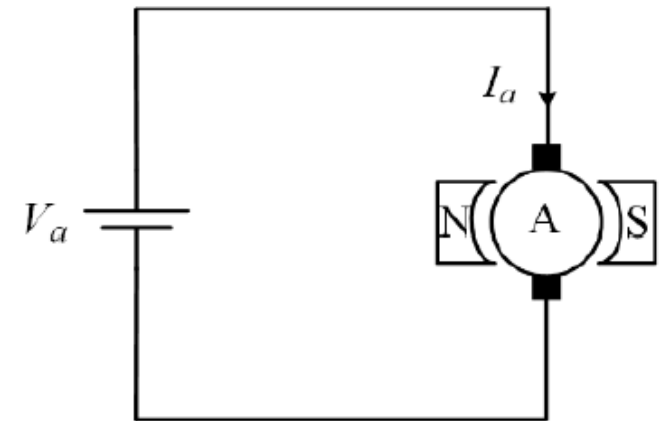
Differential compound



Series



Cumulative compound



PM

Current direction here indicates a motor, inversed direction would be a generator.

Wound Field DC Machine

❑ Separately excited

The field and armature voltages can be controlled independently.

❑ Shunt

The field and armature are connected to a common source. Independent control of the field current or armature voltage can be done only by inserting a resistance in the appropriate circuit.

❑ Series

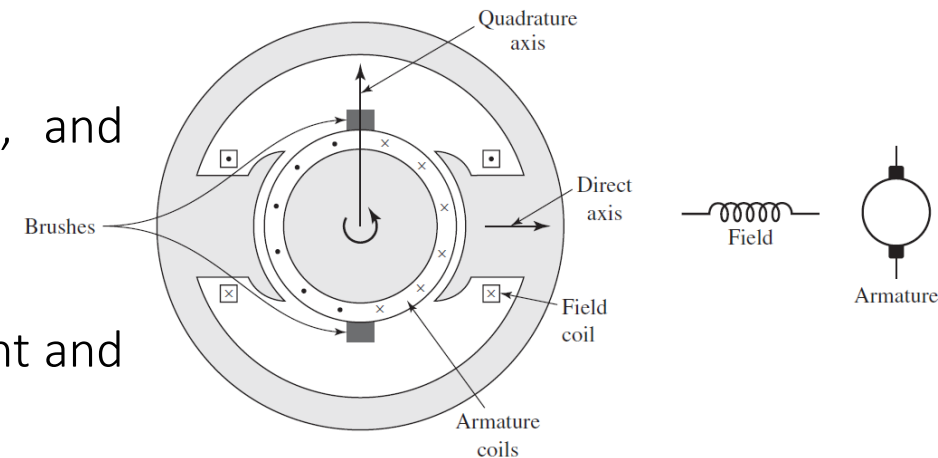
The field current is the same as the armature current, and therefore field flux is a function of armature current.

❑ Cumulative compound

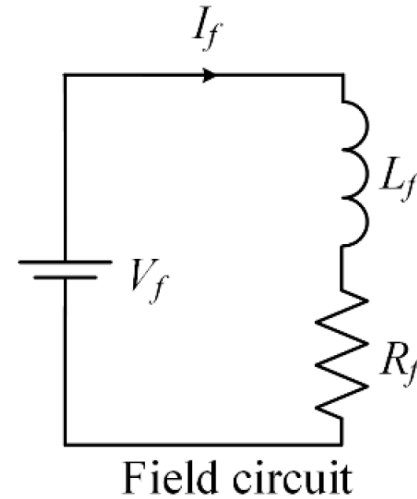
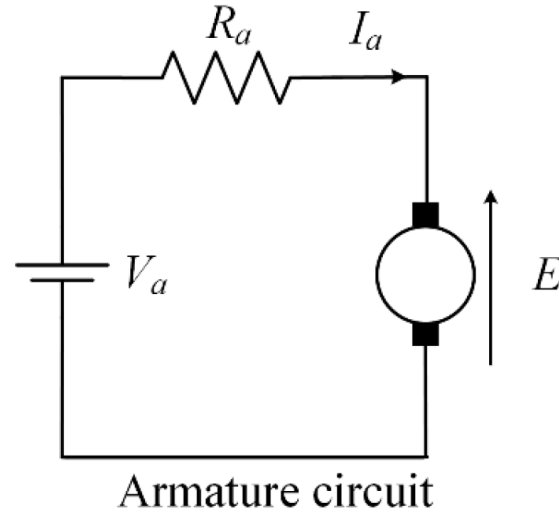
The mmf of a series field is a function of the armature current and is the same direction as the mmf of the shunt field.

❑ Differential compound

The mmf of a series field is a function of the armature current and is in opposite direction as the mmf of the shunt field.



Basic Calculations



□ Back emf

$$E = K_e \Phi \omega_m$$

□ Armature Voltage V_a

$$V = E \pm R_a I_a$$

where + is for motor and – is for generator

Basic Calculations

□ Torque $T = K_e \Phi I_a$

□ Flux $\Phi = K_f I_f$

where K_f is constant at no magnetic saturation.

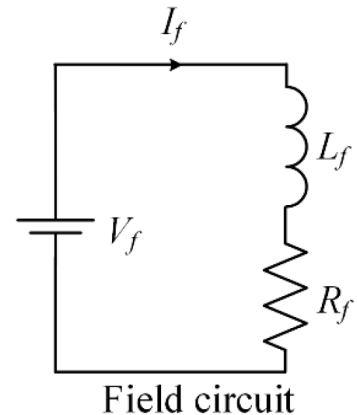
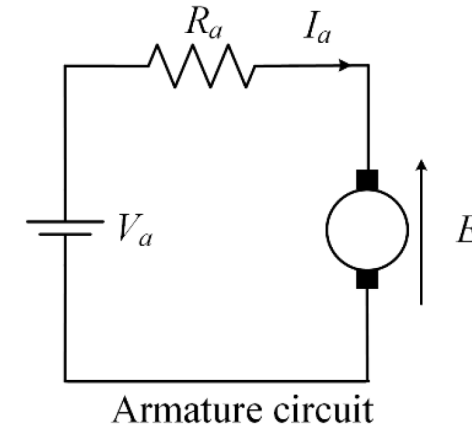
□ Motor output / generator input power

$$P = T\omega_m$$

□ Generator output / motor input power

$$P = VI_a$$

Notice that ω_m is in rad/sec, hence K_e represents both the back emf constant and torque constant.

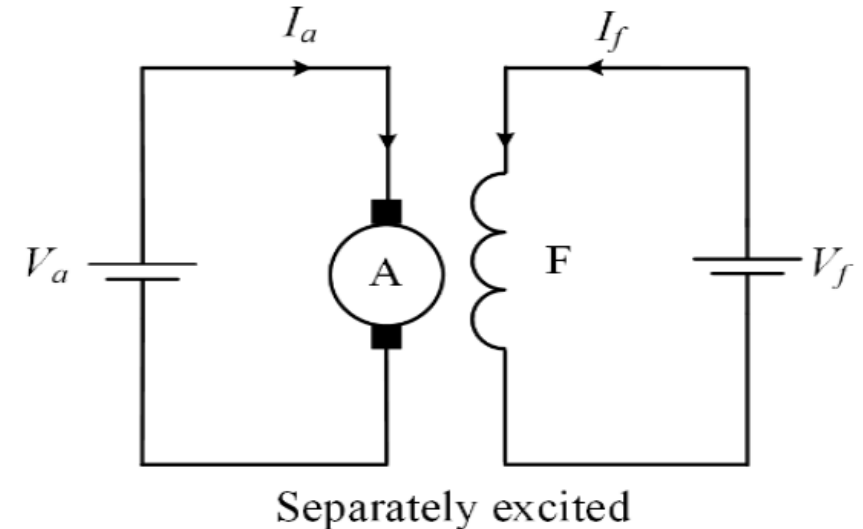


Torque-speed Characteristic of Separately Excited DC Machine

- For the separately excited DC machine, the armature and field voltages are kept at the rated value. It is actually equivalent to the shunt DC machine and PM DC machine at the rated flux. The corresponding torque-speed characteristic can be derived as:

$$T = \frac{K_e \phi V_a}{R_a} - \frac{K_e^2 \phi^2 \omega_m}{R_a}$$

- which indicates that the speed **decreases linearly** as the speed regulation depends on the armature circuit



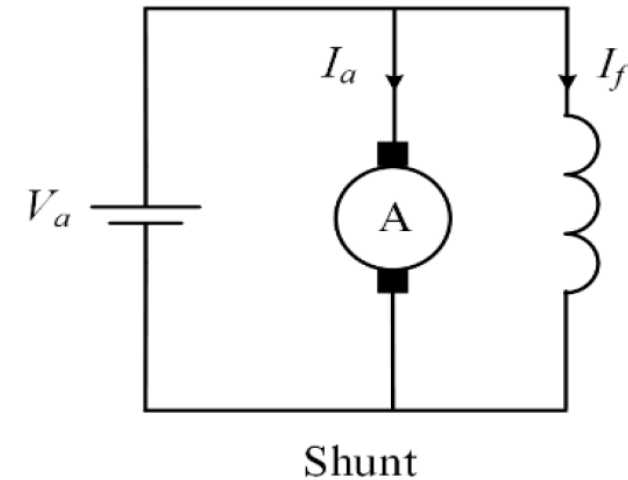
Torque-speed Characteristic of Shunt DC Machine

$$\frac{V - E}{R_a} = I_a$$

$$E = K_e \phi \omega_m$$



$$\begin{aligned} T &= K_e \phi I_a \\ &= K_e \phi \left(\frac{V - E}{R_a} \right) \\ &= \frac{K_e \phi V}{R_a} - \frac{K_e \phi E}{R_a} \\ &= \frac{K_e \phi V}{R_a} - \frac{K_e^2 \phi^2 \omega_m}{R_a} \end{aligned}$$

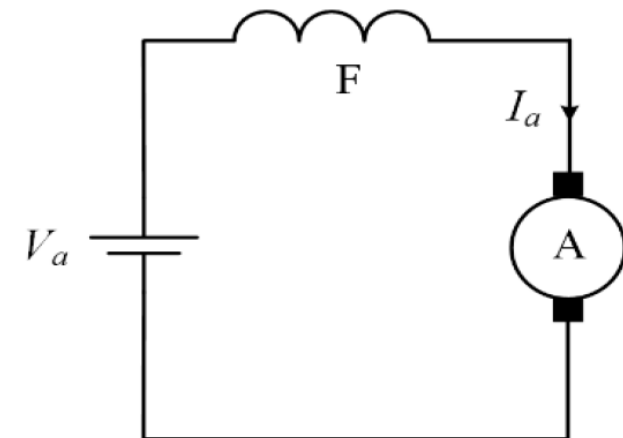


Torque-speed Characteristic of Series DC Machine

- ❑ For the series DC machine, the corresponding torque-speed characteristic can be expressed as:

$$T = \frac{K_e K_f V_a^2}{(R_a + K_e K_f \omega_m)^2}$$

- ❑ which indicates that the **speed inversely** relates to the torque.



Derivation for Series DC Machine

$$\frac{V_a - E}{R_a} = I_a$$

$$E = K_e \phi \omega_m$$

$$I_a = \frac{\phi}{K_f}$$



$$T = K_e \phi I_a$$

$$= K_e K_f I_a^2$$

$$T = K_e K_f \frac{\phi^2}{K_f^2}$$

$$= \frac{K_e}{K_f} \phi^2$$

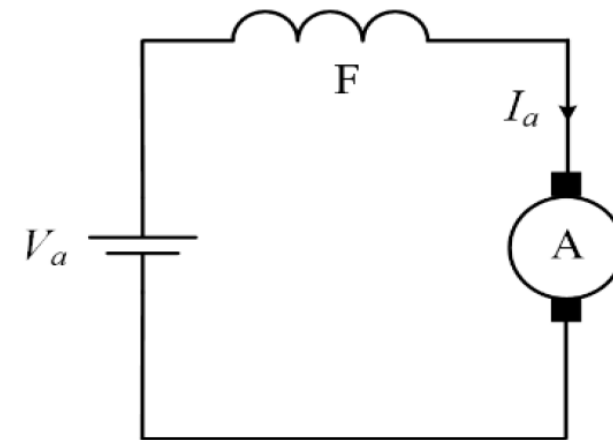
$$\phi = \sqrt{\frac{TK_f}{K_e}}$$



$$T = \frac{K_e \phi V_a}{R_a} - \frac{K_e^2 \phi^2 \omega_m}{R_a}$$

$$= \frac{K_e \sqrt{\frac{TK_f}{K_e}} V_a}{R_a} - \frac{K_e^2 \frac{TK_f}{K_e} \omega_m}{R_a}$$

$$R_a T = K_e \sqrt{\frac{TK_f}{K_e}} V_a - K_e K_f T \omega_m$$



Series



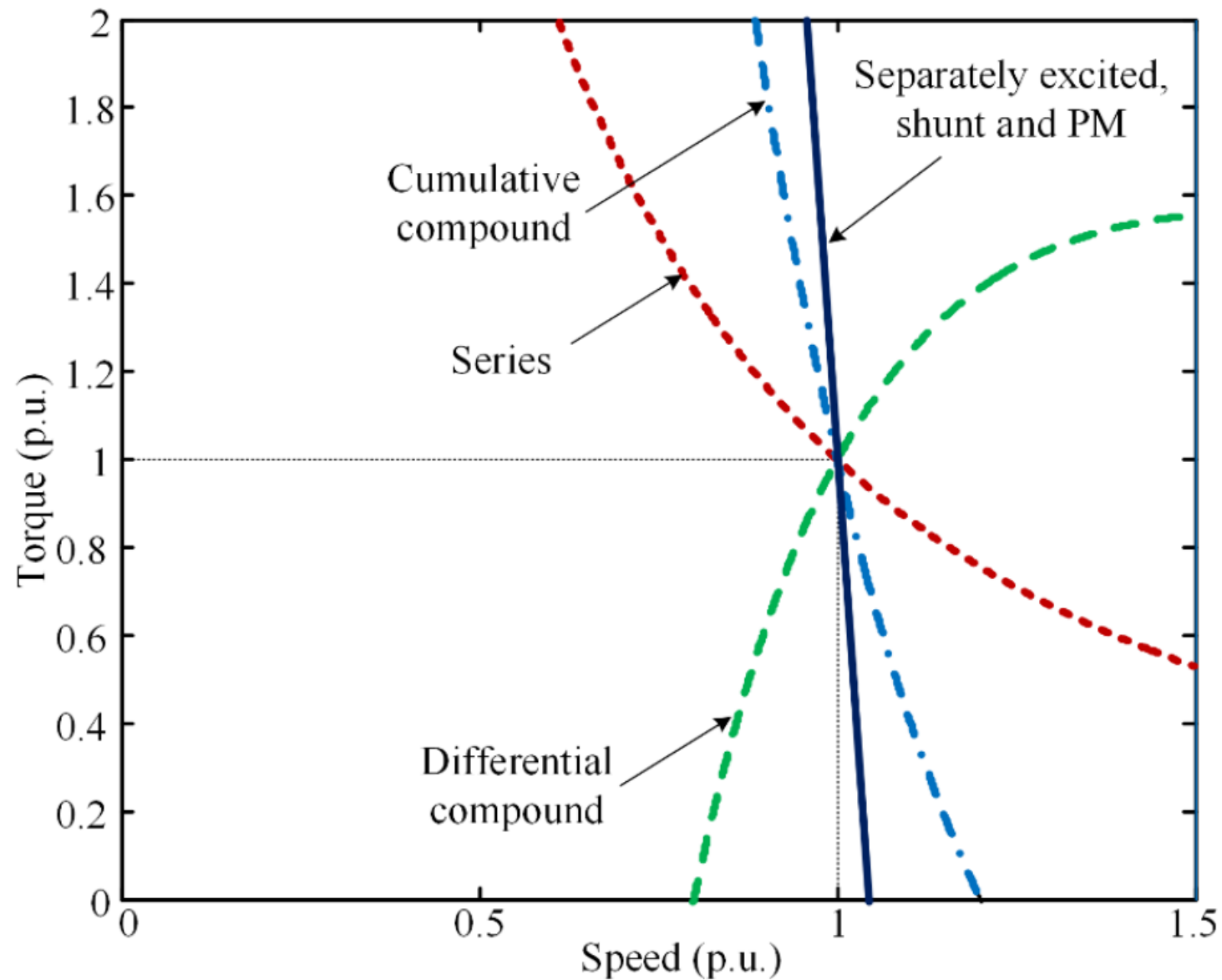
$$T(R_a + K_e K_f \omega_m) = K_e \sqrt{\frac{TK_f}{K_e}} V_a$$

$$T^2 (R_a + K_e K_f \omega_m)^2 = K_e T K_f V_a^2$$



$$T = \frac{K_e K_f V_a^2}{(R_a + K_e K_f \omega_m)^2}$$

Torque-Speed Curves



Example 1: Calculate the Torque

□ Given a 220 V, 5.15 kW DC series motor, and the motor is directly coupled to a fan. This DC series motor draws 25 A and runs at 300 rpm at terminal voltage of 220 V with no external resistance. The torque of the fan is proportional to the square of its rotating speed, if $R_a=0.6\ \Omega$ and $R_f=0.4\ \Omega$, and we neglect rotational losses. Please find:

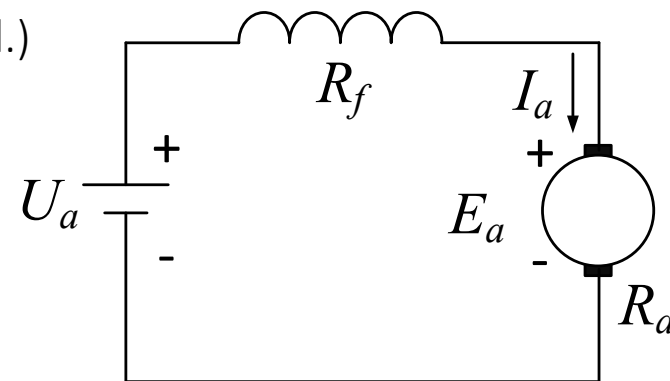
- Determine the power supplied to the fan and torque developed by the motor.
- If the rotating speed of the fan is to be reduced to 200 rpm by inserting an external resistance, determine the value of this resistance and the power delivered to the fan.

(Hint1: Terminal voltage is the voltage measured across DC machine's terminals.

Hint2: Terminal voltage does not change before and after the rotating speed variation.

Hint 3: You may need to use the relation between the back EMF and the field current within the solution process.

Hint 4: Wind resistance force is proportional to the square of fan's speed.)



Example 1: Calculate the Torque

Solution

(a) First, we should list all the known conditions for the question and draw the equivalent circuit of the DC series motor:

$$U_a = 220 \text{ V}, \quad P_{\text{out_rated}} = 5.15 \text{ kW}, \quad I_{a1} = 25 \text{ A}, \quad N1 = 300 \text{ rpm} \\ T \propto N^2, \quad R_a = 0.6 \text{ } \Omega, \quad R_f = 0.4 \text{ } \Omega$$

Thus, the back EMF can be calculated as:

$$E_a = U_a - I_{a1}(R_a + R_f) = 220 - 25 \cdot 1 = 195 \text{ V}$$

Since the rotational loss is ignored, the output power can be calculated as the induced EMF multiple the current:

$$P_{\text{out}} \approx P_d = E_a \cdot I_{a1} = 195 \cdot 25 = 4.875 \text{ kW}$$

As for the torque, it can be acquired based on the relation between power and rotating speed:

$$P_{\text{out}} = T \cdot \omega \Rightarrow T = \frac{P_{\text{out}}}{\omega} = 155.18 \text{ Nm}$$

Example 1: Calculate the Torque

Solution

(b) After the rotating speed is reduced to 200 rpm, the new torque can be calculated based on the relation between the torque and the rotating speed:

$$T \propto N^2 \Rightarrow \frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^2 \quad \longrightarrow \quad T_2 = \left(\frac{N_2}{N_1}\right)^2 T_1 = \left(\frac{200}{300}\right)^2 \cdot 155.18 = 68.97 \text{ Nm}$$

For series DC motor, $I_f = I_a$. Thus, the relation between the induced EMF and field current can be expressed as:

$$E_a = K_e \Phi \omega = K_e K_f I_f \omega \Rightarrow \frac{E_2}{E_1} = \frac{I_{f2} N_2}{I_{f1} N_1}$$

Then, we have:

$$\frac{E_2}{195} = \frac{I_{a2} \cdot 200}{25 \cdot 300} \Rightarrow E_2 = 5.2 I_{a2}$$

Based on the relation between power, torque, and rotating speed:

$$T_2 = \frac{E_2 I_{a2}}{N_2} \Rightarrow 68.97 = \frac{5.2 \cdot I_{a2}^2}{20.94}$$

Hence, we can obtain: $I_{a2} = 16.67 \text{ A}$

Based on the relation between terminal voltage, the external resistance can be calculated as:

$$E_2 = U_a - I_{a2}(R_a + R_f + R_{ext}) \Rightarrow 5.2 \cdot 16.67 = 220 - 16.67 \cdot (1 + R_{ext})$$

Therefore, $R_{ext} = 7 \Omega$. At this moment, the output power can be calculated as:

$$P_{out2} \approx P_{d2} = E_2 \cdot I_{a2} = 5.2 \cdot (16.67)^2 = 1.445 \text{ kW}$$

Speed Control of DC Motors

How to control Speed ?

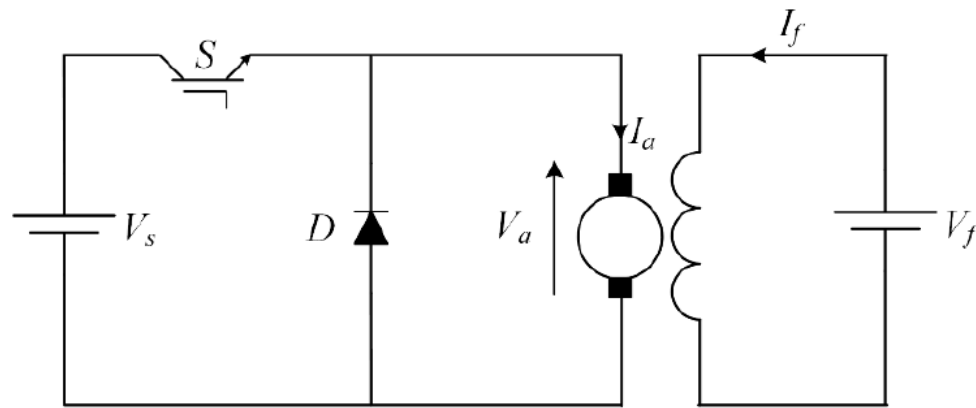


Speed Control of DC Motors

- ☐ Armature voltage control
 - ☐ Field flux control
 - ☐ Combined armature voltage & field flux control
 - ☐ Armature resistance control (less common method)
-
- ☐ Torque-speed characteristics of Compounded DC Motors
 - ☐ Problem on Starting
 - ☐ Losses in a DC motor

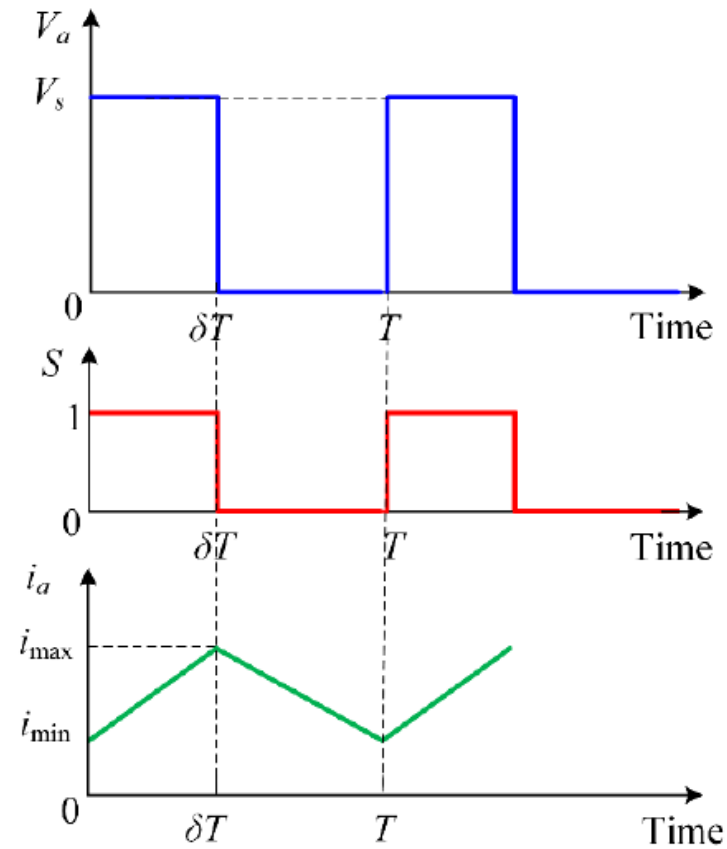
Armature Voltage Control

❑ Separately excited and shunt DC motors



$$V_a = \frac{1}{T} \int_0^T v_a dt = \frac{1}{T} \int_0^{\delta T} V_s dt = \delta V_s$$

$$I_a = \frac{V_a - E}{R_a}$$



Armature Voltage Control

□ Summarizing the cause-and-effect behavior

1. An increase in V_a increase $I_a = \frac{V_a \uparrow - E_a}{R_a}$

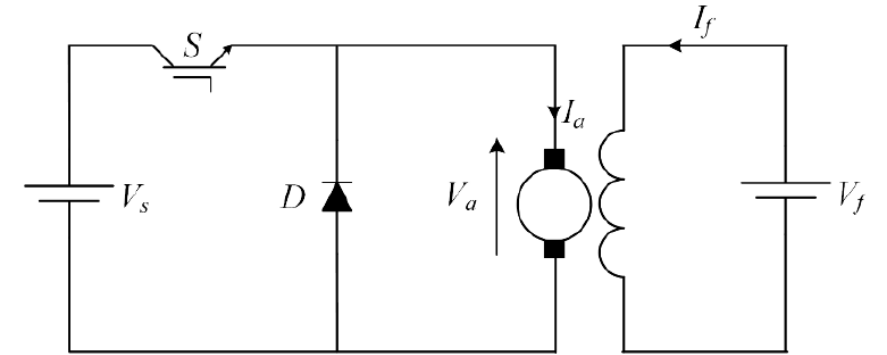
2. Increasing I_a increase induced torque $\tau_{ind} = K_e \phi I_a \uparrow$

3. Increasing τ_{ind} makes $\tau_{ind} > \tau_{load}$, ω increasing.

4. Increasing ω increases $E_a = K_e \phi \omega \uparrow$

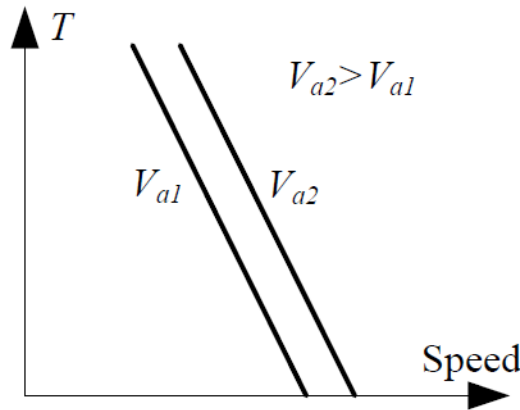
5. Increasing E_a decreases $I_a = \frac{V_a - E_a \uparrow}{R_a}$

6. Decreasing I_a decreases τ_{ind} until $\tau_{ind} = \tau_{load}$ at a higher ω .

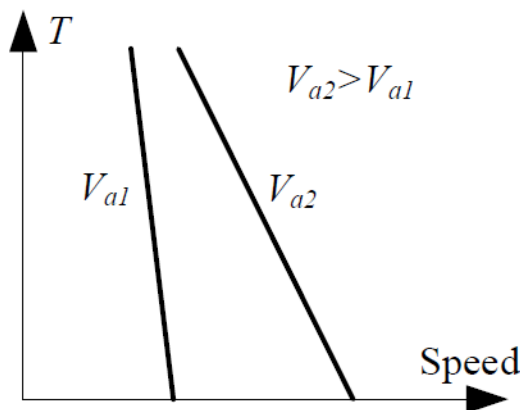


Armature Voltage Control

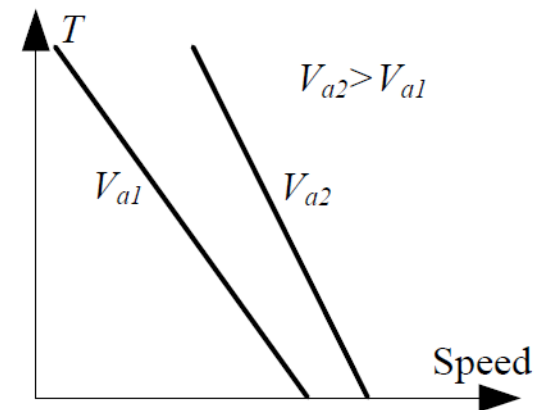
- When the armature voltage is reduced from V_{a2} to V_{a1} , which slope of the torque-speed characteristic is correct ?



A
Constant



B
Increase



C
Reduce

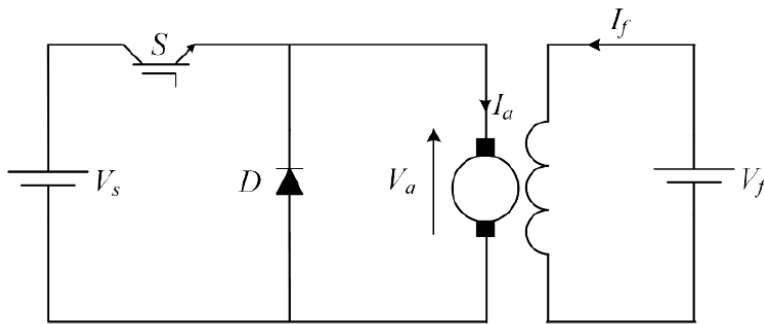
Armature Voltage Control

□ Meaning of the equation

$$T = \frac{K_e \phi V_a}{R_a} - \frac{K_e^2 \phi^2 \omega_m}{R_a}$$

$$T = -\frac{K_e^2 \phi^2}{R_a} \omega_m + \frac{K_e \phi V_a}{R_a}$$

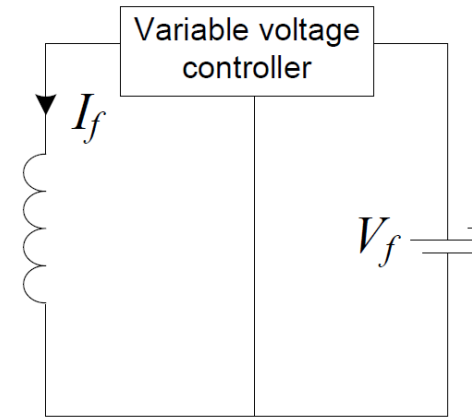
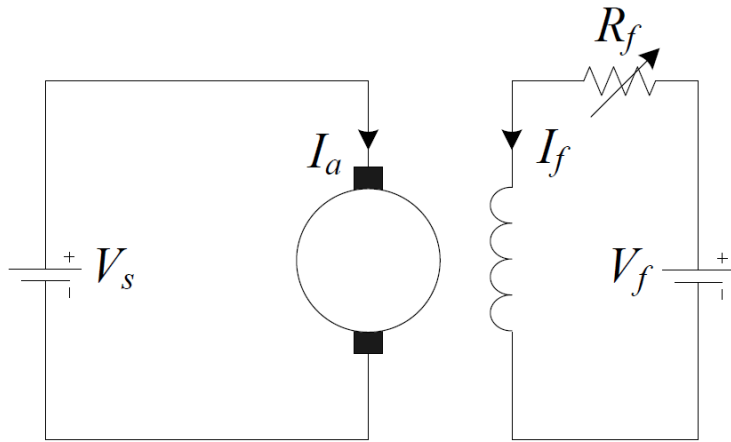
$$Y = m X + C$$



- If the armature voltage is reduced, the armature current and hence the motor torque and the motor speed will decrease. The nature and the slope of the speed-torque characteristics do not change with the change in speed.
- The maximum torque capability of the motor remains unchanged at all the speeds because of the maximum permissible armature current.
- Controlling the voltage to regulate the armature current provides a constant torque drive regardless of the motor speed since the magnetic flux is constant.

Field Flux Control

- Separately excited and shunt DC motors



What happens when the field voltage of a dc motor is changed ?

- Torque $T = K_e \Phi I_a$

- Back emf $E = K_e \Phi \omega_m$

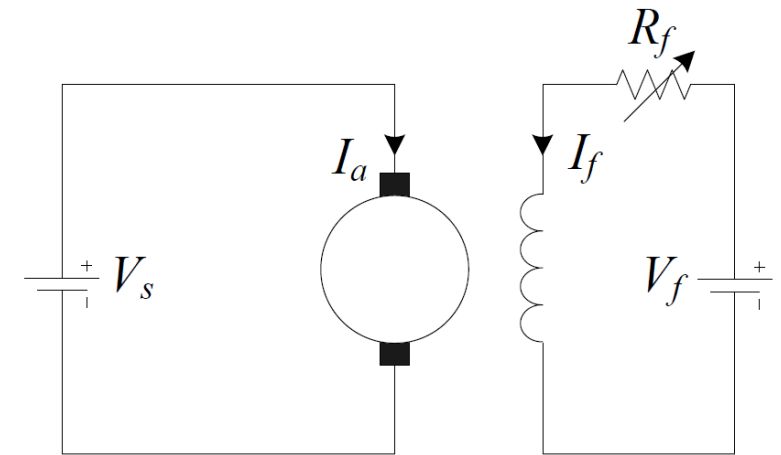
- Flux $\Phi = K_f I_f$

- Armature Voltage $V = E \pm R_a I_a$

Field Flux Control

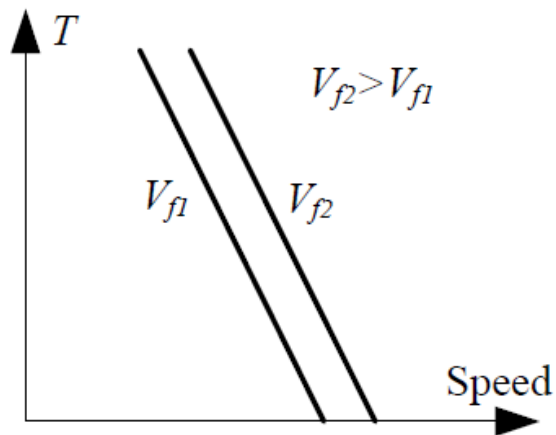
□ Summarizing the cause-and-effect behavior

1. Decreasing V_f causes $I_f = \frac{V_f \downarrow}{R_f}$ to decrease
2. Decreasing I_f decreases ϕ and lowers $E_a = K_e \phi \downarrow \omega$
3. Decreasing E_a increases $I_a = \frac{V_a - E_a \downarrow}{R_a}$
4. Increasing I_a increases $\tau_{ind} = K_e \phi \downarrow I_a \uparrow$
5. Increasing τ_{ind} makes $\tau_{ind} > \tau_{load}$, speed ω increases.
6. Increasing ω increases $E_a = K_e \phi \omega \uparrow$
7. Increasing E_a decreases $I_a = \frac{V_a - E_a \uparrow}{R_a}$
8. Decreasing I_a decreases τ_{ind} until $\tau_{ind} = \tau_{load}$ at a higher ω

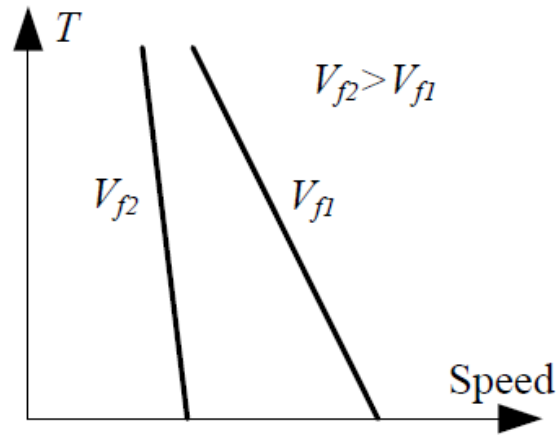


Field Flux Control

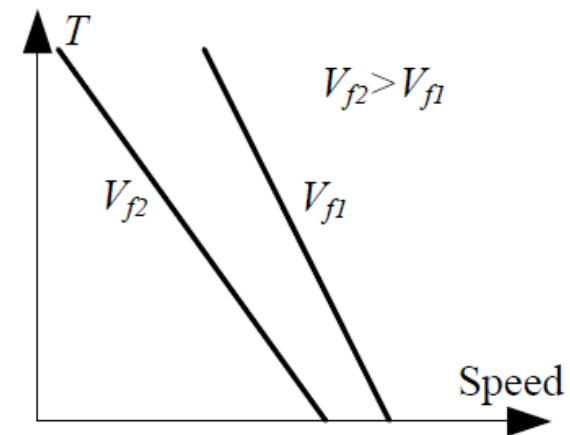
- ❑ When the field voltage is reduced from V_{f2} to V_{f1} , which slope of the torque-speed characteristic is correct ?



A
Constant



B
Decrease



C
Increase

Field Flux Control

- ❑ If the field is weakened, the induced *emf* will decrease. Because of low armature resistance, the current increases by an amount much larger than the decrease in the field flux. Thus, the motor torque and hence the motor speed will increase.
- ❑ As the flux in the machine decreases, the no-load speed of the motor increases, while the slope of the torque-speed curve becomes gentle.

Example 2: Field Flux Control

□ A separately excited DC motor with $K_e\Phi$ of 1 and R_a of $0.25\ \Omega$. It is currently operating with a terminal voltage of 250 V and an internal generated voltage of 245 V. What is the no-load speed of this motor? What happens in this motor if there is a 10 percent decrease in flux?

A. 250 rad/s ($K_e\Phi=1$), 277 rad/s ($K_e\Phi=0.9$)

B. 245 rad/s ($K_e\Phi=1$), 250 rad/s ($K_e\Phi=0.9$)

C. 245 rad/s ($K_e\Phi=1$), 277 rad/s ($K_e\Phi=0.9$)

Example 2: Field Flux Control

□ Solution

- 1) What is the no-load speed of this motor

$$T = \frac{K_e \phi V}{R_a} - \frac{K_e^2 \phi^2 \omega_m}{R_a}$$

$$\frac{K_e^2 \phi^2 \omega_m}{R_a} = \frac{K_e \phi V}{R_a}$$

$$\begin{aligned}\omega_m &= V \\ &= 250\end{aligned}$$

- 2) A 10 percent decrease in flux

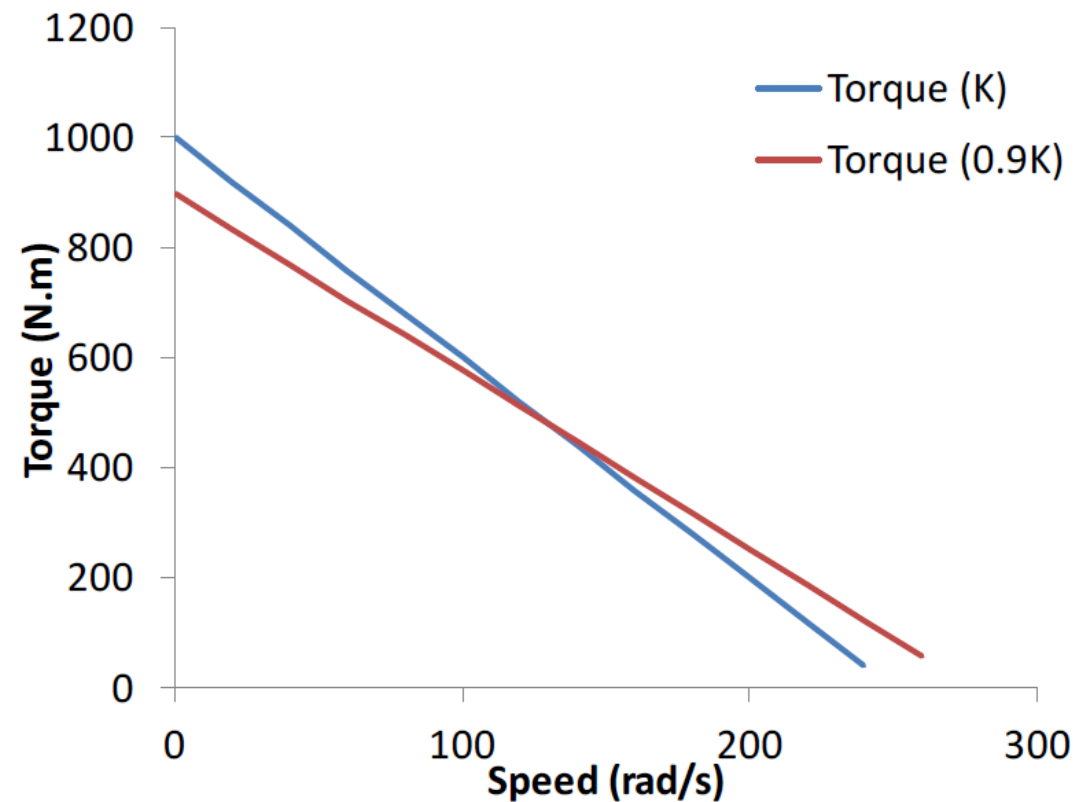
$$\frac{K_e^2 \phi^2 \omega_m}{R_a} = \frac{K_e \phi V}{R_a}$$

$$0.81 \omega_m = 0.9V$$

$$\omega_m = 277.777$$

Field Flux Control

- ❑ At slow speeds a decrease in flux will actually decrease the speed of the motor.
- ❑ This effect occurs because at low speeds, the increase in armature current caused by the decrease in E_a is no longer large enough to compensate for the decrease in flux in the induced torque equation.



Combined Armature Voltage & Field Flux Control

- ❑ A motor is operating at its rated terminal voltage, power, and field current, then it will be running at rated speed (base speed).
- ❑ Armature voltage control can control the speed of the motor for speeds below base speed but not for speeds above base speed.
- ❑ To achieve a speed faster than base speed by armature voltage control would require excessive armature voltage, possibly damaging the armature circuit.
- ❑ Armature voltage control has the advantage of retaining the maximum torque capability, it is employed wherever it can be.

Combined Armature Voltage & Field Flux Control

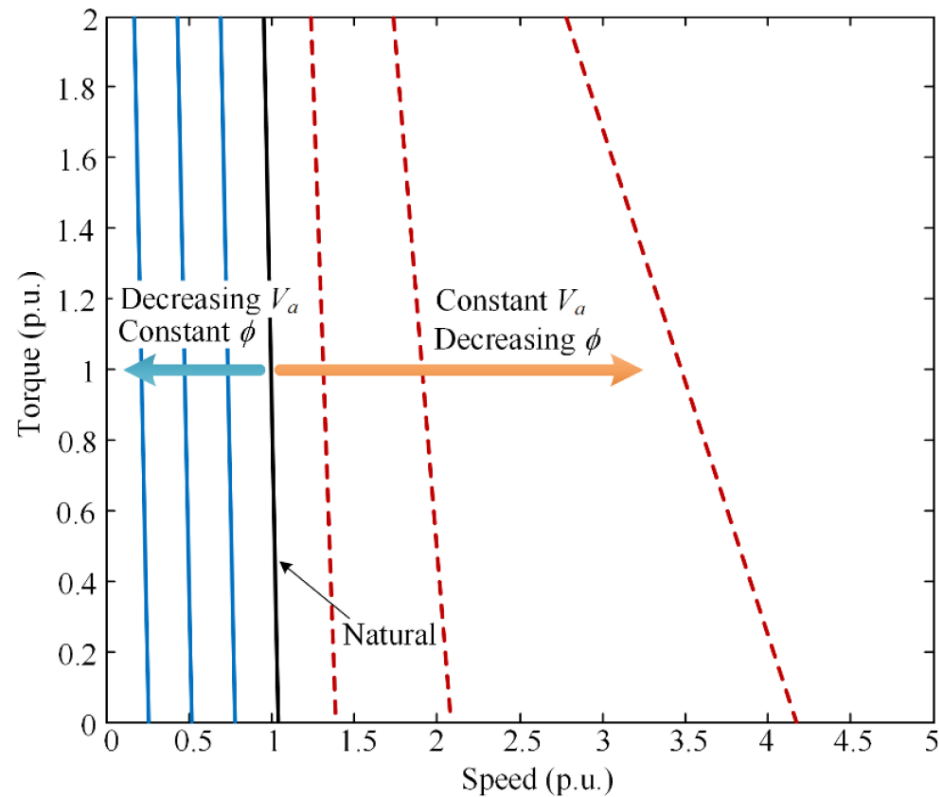
- ❑ A motor is operating at its rated terminal voltage, power, and field current, then it will be running at rated speed (base speed).
- ❑ Field resistance/current/voltage control can control the speed of the motor for speeds above base speed but not for speeds below base speed.
- ❑ To achieve a speed slower than base speed by field circuit control would require excessive field current, possibly burning up the field windings.
- ❑ To increase speed beyond the rated value using flux control, the load torque demand must be met even with the reduced torque capability.

Combined Armature Voltage & Field Flux Control

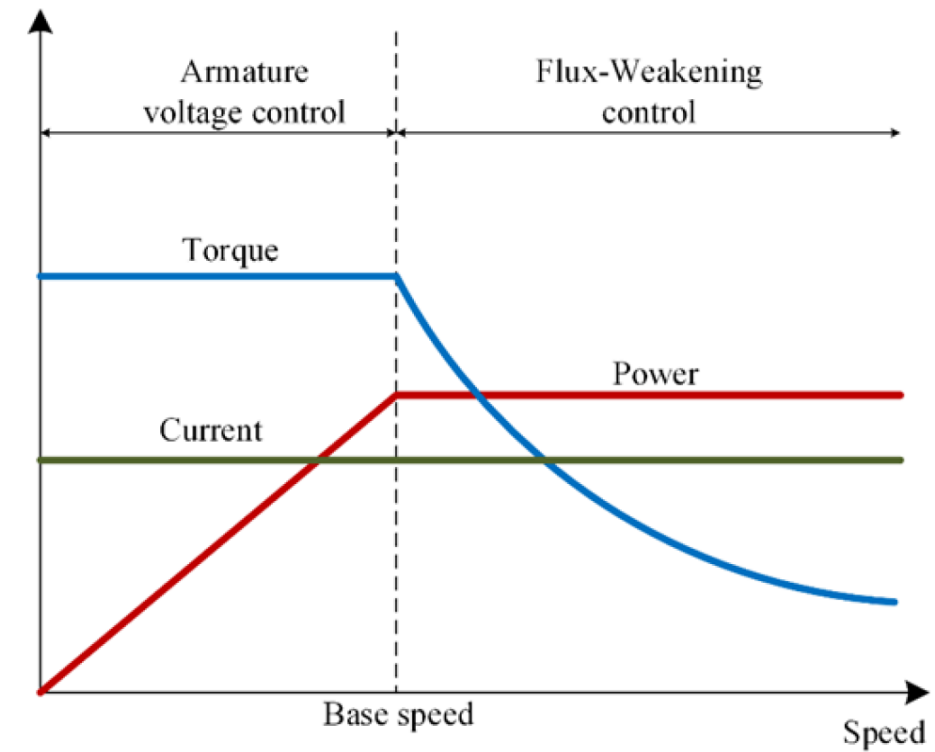
- ❑ These two techniques of speed control are obviously **complementary**. Armature voltage control works well for speeds below base speed, and field resistance/voltage/current control works well for speeds above base speed.
- ❑ By combining the two speed-control techniques in the same motor, it is possible to **get a range of speed variations** of up to 40 to 1 or more.
- ❑ Shunt and separately excited DC motors are excellent choices for applications needing **large variations in speed**, especially if the speed variations must be controlled accurately.

Combined Armature Voltage & Field Flux Control

❑ Characteristics of separately excited DC motor



❑ Capabilities of separately excited DC motor

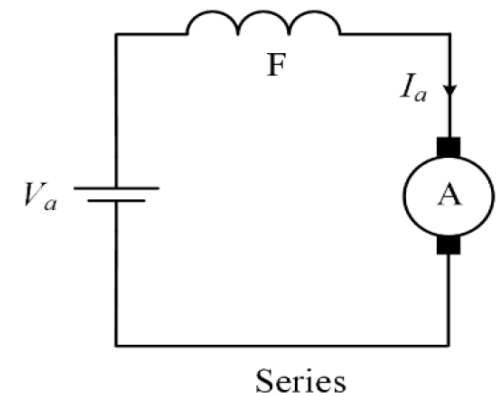


Terminal (Armature) Voltage Control

- ❑ There is only one efficient way to change the speed of a series DC motor. That method is to change the terminal voltage of the motor.
 - ❑ The speed of series DC motors can also be controlled by the insertion of a series resistor into the motor circuit, but this technique is very wasteful of power.
- Observing the torque-speed equation, predict what happen if the load on series motor goes to zero?

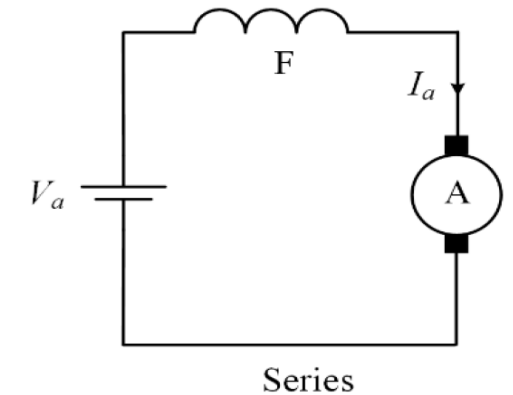
$$T = \frac{K_e K_f V_a^2}{(R_a + K_e K_f \omega_m)^2}$$

- A. Speed goes to infinity
- B. Speed goes to zero
- C. Speed keeps a rated



Terminal (Armature) Voltage Control

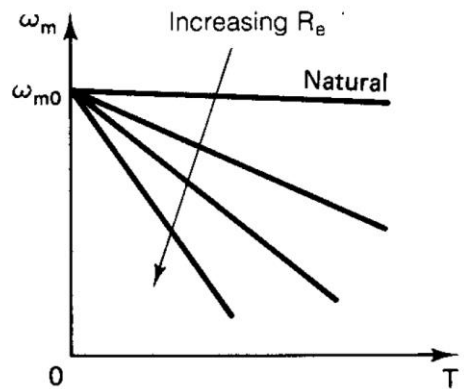
- ❑ Caution! The torque on series motor goes to zero, its speed goes to infinity.
- ❑ Never unload a series motor, and never connect one to a load by a belt or other mechanism that could break. If that were to happen and the motor were to become unloaded while running, the results could be serious.
- ❑ Advantage: The torque in the series DC motor is proportional to the square of its armature current. As a result of this relationship, it gives more torque per ampere than any other DC motor.
- ❑ They were used in applications requiring very high torques, eg. traction. Nowadays we use induction motor /w inverter for bulk traction loads (like train) and BLDC (brush less DC motor) for light traction loads.



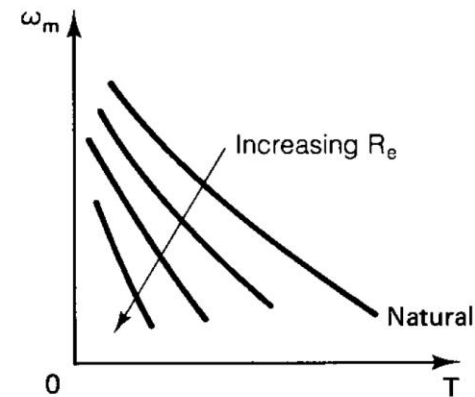
$$T = \frac{K_e K_f V_a^2}{(R_a + K_e K_f \omega_m)^2}$$

Armature Resistance Control

- ❑ Speed torque characteristics can be controlled by connecting external resistance R_e in series with the armature. The main drawback is its poor efficiency.



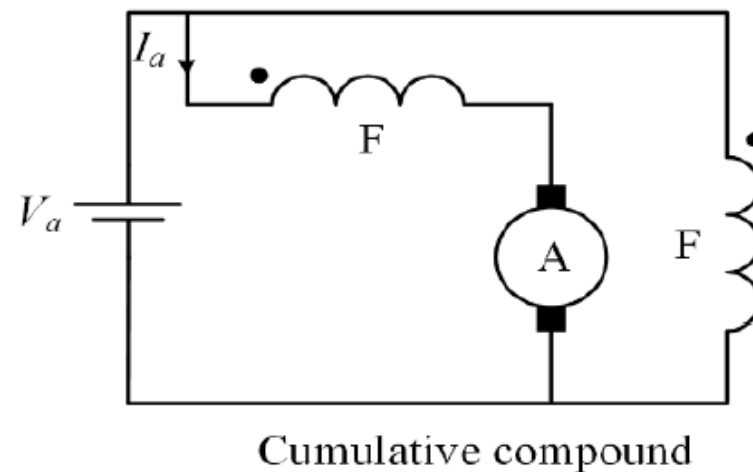
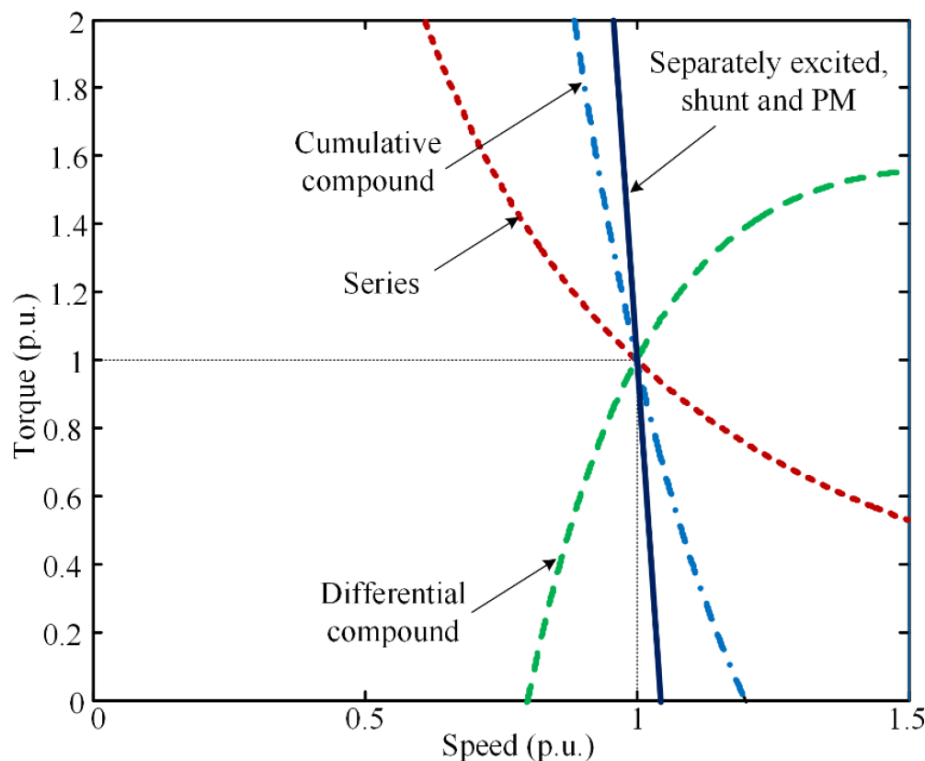
Characteristics of separately excited DC motor



Characteristics of series DC motor

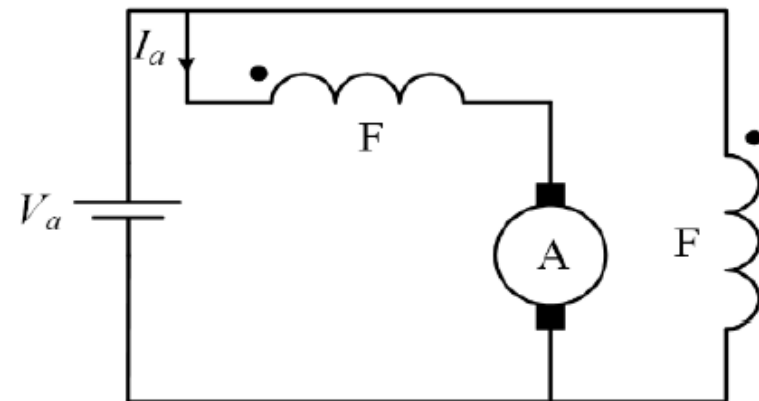
Cumulatively Compounded DC Motor

- ❑ A component of flux which is constant and another component which is proportional to its armature current.
- ❑ At light loads, the series field has a very small effect. It likes a shunt motor.
- ❑ At heavy loads, the series flux becomes important. The torque-speed curve begins to look like a series motor.



Cumulatively Compounded DC Motor

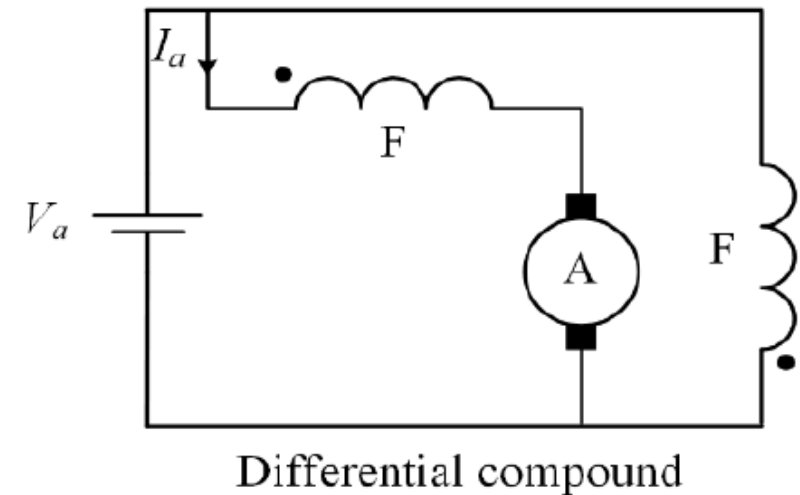
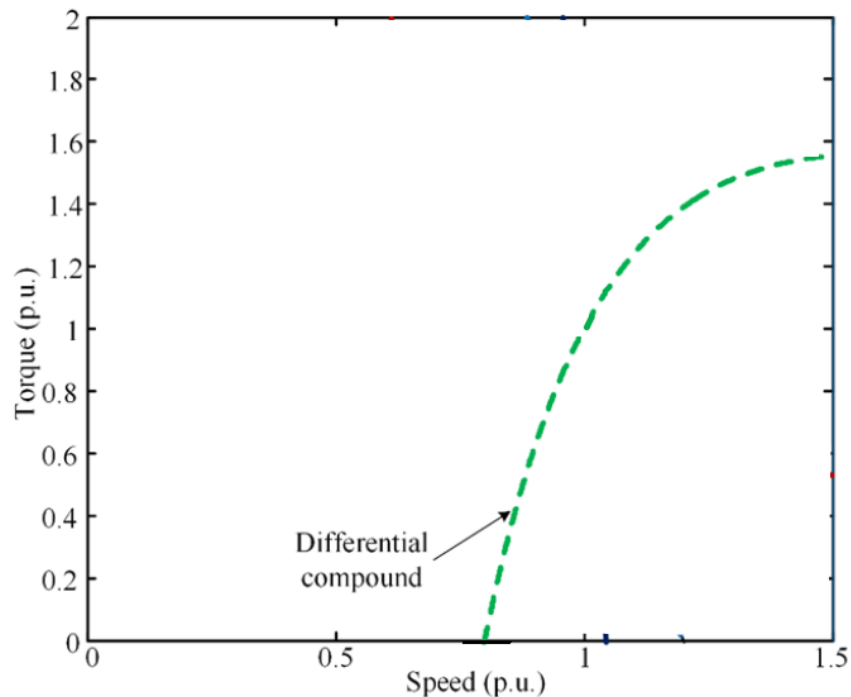
- ❑ It has higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series (whose entire flux is proportional to armature current).
- ❑ It combines the best features of both the shunt and series motors. Like a series motor, it has extra torque for starting; like a shunt motor, it does not over speed at no load.



Cumulative compound

Differentially Compounded DC Motor

- ❑ Shunt magnetomotive force and series magnetomotive force subtract from each other.
- ❑ Load on the motor increases, I_a increases and the flux decreases
- ❑ It is unstable and tends to run away.
- ❑ It is so bad that this motor is unsuitable for almost all applications.



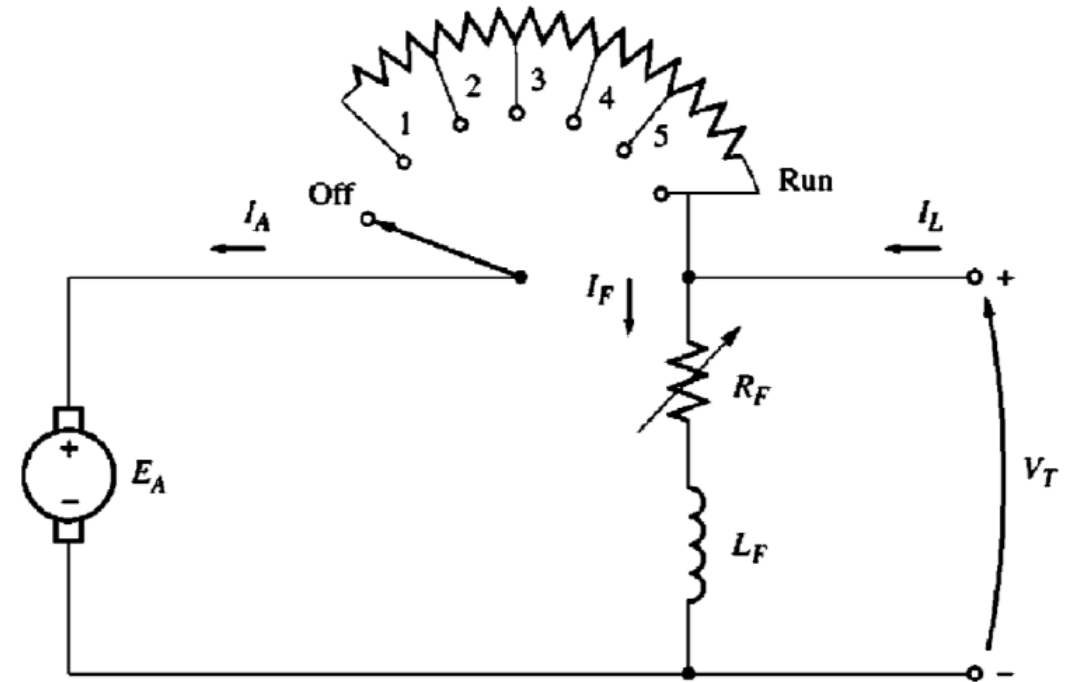
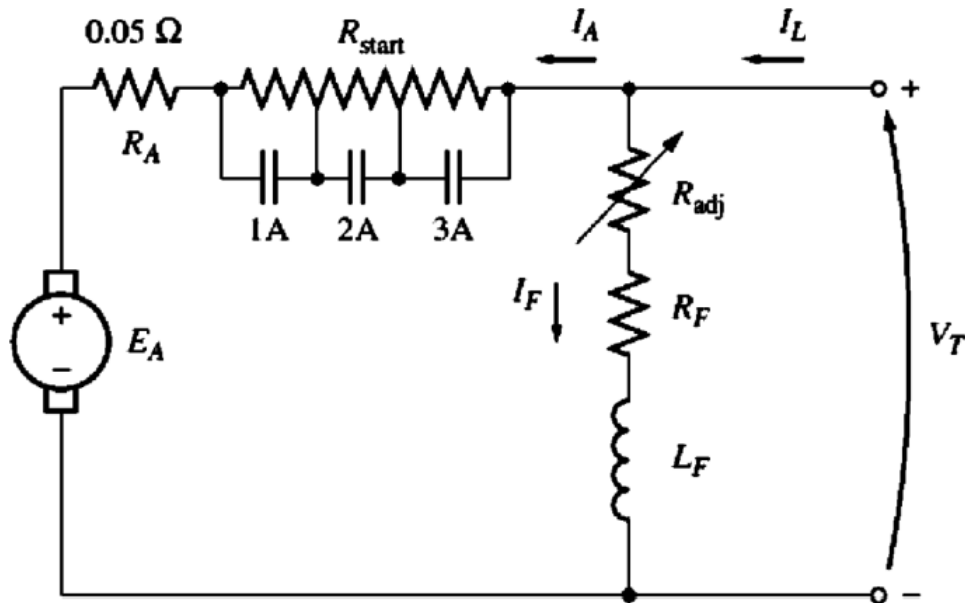
Differentially Compounded DC Motor

- ❑ At starting conditions, the motor is not turning, and so $E_a = 0V$.
- ❑ The internal resistance of a normal DC motor is very low (3-6% per unit)
- ❑ A 50 hp 250V motor, the full-load current is less than 200A, however the current on starting is up to 4167A! (over 20 times)
- ❑ Solution: using power electronic drive to control the armature current Or insert a starting resistor in series with armature to limit the current.

DC Motor Starter

❑ (Left) A shunt motor with a starting resistor in series with its armature.

❑ (Right) A manual DC motor starter.



Losses in a DC motor

- ❑ Copper losses,

 - I^2R losses in the armature and field windings

- ❑ Brush drop losses

 - Product of the brush voltage drop VBD and the armature current I_a

- ❑ Mechanical losses

 - No-load rotational losses of the motor.

- ❑ Core losses

 - Usually determined together with mechanical losses

- ❑ Stray losses

 - Losses that cannot be easily accounted. eg. short-circuit currents during commutation. Typical $<1\%$ (usually approximation, cannot be calculated accurately)

Example 3: DC Motor Efficiency Calculations

□ A 50-hp 250V 1200 r/min shunt DC motor has a rated armature current of 170 A and a rated field current of 5 A. When its rotor is blocked, an armature voltage of 10.2 V produces 170 A of current flow, and a field voltage of 250 V produces a field current flow of 5 A. The brush voltage drop is assumed to be 2 V. At no load with the terminal voltage equal to 240 V, the armature current is equal to 13.2 A, the field current is 4.8 A, and the motor's speed is 1150 r/min.

1. Determine the armature losses (P_a), field losses (P_f) and rotational losses (P_{rot}) when armature current is 170 A. Which set of answer is correct?

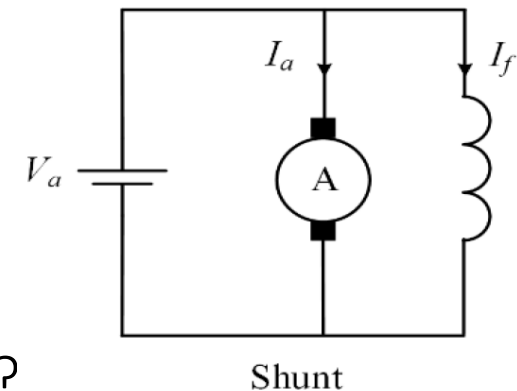
A. 1,734 W, 1,250 W and 3,158 W

B. 1,734 W, 1,200 W and 3,158 W

C. 2,074 W, 1,250 W and 3,331 W

2. How much power is output from this motor at rated conditions?

3. What is the motor's efficiency?



Example 3: DC Motor Efficiency Calculations

□ Solution

Armature resistance and losses

$$R_a = \frac{10.2 \text{ V}}{170 \text{ A}} = 0.06 \Omega \quad P_a = (170 \text{ A})^2 (0.06 \Omega) = 1734 \text{ W}$$

Field resistance and losses

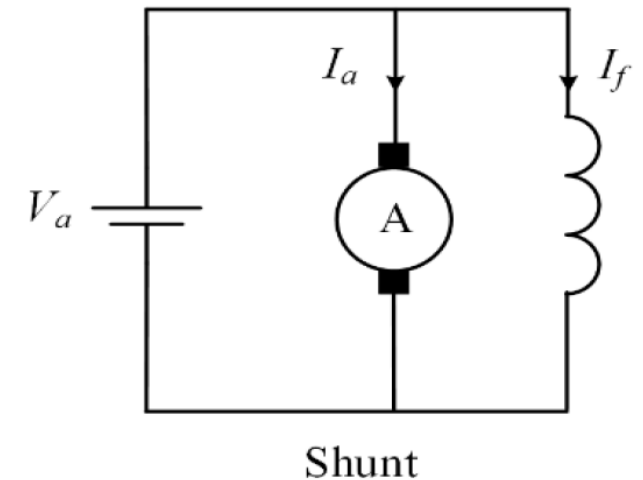
$$R_f = \frac{250 \text{ V}}{5 \text{ A}} = 50 \Omega \quad P_f = (5 \text{ A})^2 (50 \Omega) = 1250 \text{ W}$$

Brush losses at full load

$$P_{brush} = (2 \text{ V}) 170 \text{ A} = 340 \text{ W}$$

Rotational losses

$$P_{rot} = P_{core} + P_{mech} = (240 \text{ V})(13.2 \text{ A}) - 13.2 * 13.2 * 0.06 = 3158 \text{ W}$$



Example 3: DC Motor Efficiency Calculations

□ Solution

Input power of the motor $P_{in} = (250 \text{ V})(175 \text{ A}) = 43750 \text{ W}$

Output power
$$\begin{aligned} P_o &= P_{in} - P_{brush} - P_{cu} - P_{core} - P_{mech} - P_{stray} \\ &= 43750 - 340 - 1734 - 1250 - 3158 - (0.01)(43750) \\ &= 36,830 \text{ W} \end{aligned}$$

Efficiency
$$\begin{aligned} \eta &= \frac{P_{out}}{P_{in}} \times 100\% \\ &= \frac{36,830}{43,750} = 84.2\% \end{aligned}$$

